

【ヒルベルト変換 Hilbert transform】

理想 Hilbert 変換器 周波数応答: $H(\omega) = -j\text{sgn}(\omega)$

但し,

$$\text{sgn}(\omega) = \begin{cases} 1 & (\omega > 0) \\ -1 & (\omega < 0) \end{cases}$$

ここで, sgn 関数の fourier 変換は,

$$F[\text{sgn}(t)] = \int_{-\infty}^{+\infty} \text{sgn}(t)e^{-j\omega t} dt = \frac{2}{j\omega}$$

である. fourier 変換の双対性より, $F[\frac{2}{jt}] = 2\pi\text{sgn}(-\omega) = -2\pi\text{sgn}(\omega)$ となる.

※ sgn 関数は奇関数

ここから,

$$F[\frac{1}{\pi t}] = -j\text{sgn}(\omega)$$

ゆえに, インパルス応答 $h(t)$ は,

$$h(t) = \frac{1}{\pi t}$$

となる.

入力信号 $x(t)$ を Hilbert 変換した信号を $\hat{x}(t)$ とすると,

$$\hat{x}(t) = \text{Convolution}[x(t), \frac{1}{\pi t}] = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau$$

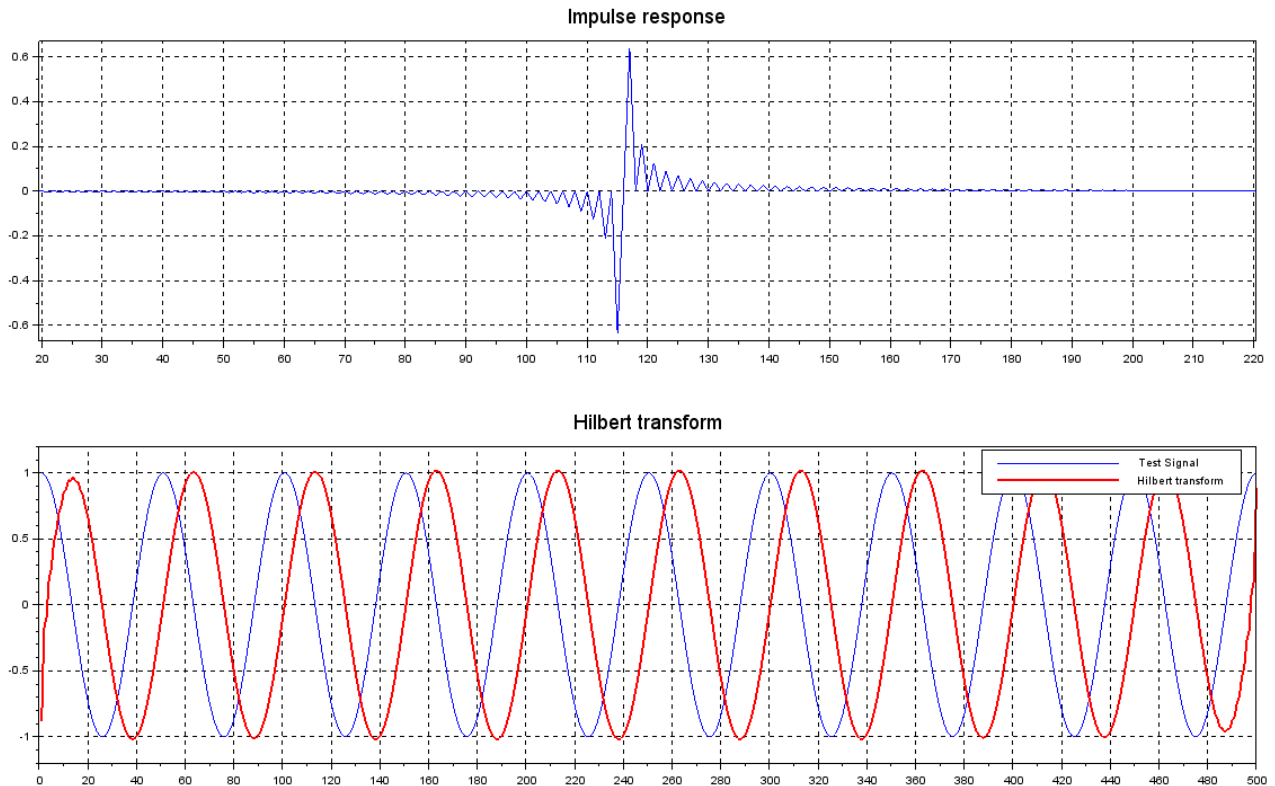


Figure 1: Scilab 実行結果

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////////////////////////////////////
//      ヒルベルト変換
//      Hilbert transform
//
//      M.Tsutsui
////////////////////////////////////

clear;

d_size=500;//データサイズ

t=linspace(0,d_size,d_size);

f=10;//周波数
omega=2*%pi*f;//角周波数
x=cos(omega*t);//Test Signal

//-----フィルタ係数_Table-----//
//次数:230
//インパルス応答 h(t)=1/πt

//下記*1を張り付け.
//-----//

x_hat=conv(x,coef,"same");//畳込み演算

subplot(2,1,1)
plot(coef);
title('Impulse_response','fontsize',4);
xgrid();

subplot(2,1,2)
plot(x);
plot(x_hat,'r');
g=gce();
c=g.children;
c.thickness=1.5;
xgrid();
hl=legend(['Test_Signal';'Hilbert_transform']);
title('Hilbert_transform','fontsize',4);

```

*1 (フィルタ係数)

```
coef=[-1.10216831e-02, -5.35235376e-08, -1.62741486e-03, 9.19962458e-07,-1.74850348e-03, -2.00519718e-06, -  
1.87219781e-03, 2.34178138e-06,-2.00601700e-03, -2.73175426e-07, -2.14448814e-03, -9.34700849e-07,-2.28754302e-  
03, -8.25998933e-07, -2.43947457e-03, 5.38942588e-07,-2.59837203e-03, 9.06487236e-07, -2.76513999e-03, -  
4.18722090e-07,-2.93876327e-03, -1.25863103e-06, -3.12055483e-03, -1.44895154e-06,-3.31097292e-03, -6.56468906e-07,  
-3.51019582e-03, -3.19115408e-07,-3.71917864e-03, -3.75253200e-07, -3.93814423e-03, -6.02714618e-07,-4.16789478e-  
03, -8.79949234e-07, -4.40826484e-03, -9.97192787e-07,-4.66000529e-03, -1.21345814e-06, -4.92412882e-03, -  
1.10763134e-06,-5.20173440e-03, -8.86491063e-07, -5.49382760e-03, -5.37789933e-07,-5.80096419e-03, -2.51355452e-07,  
-6.12438134e-03, -1.12788254e-07,-6.46524157e-03, -7.91588657e-08, -6.82509018e-03, -3.71474113e-07,-7.20536228e-  
03, -7.69769481e-07, -7.60776848e-03, -1.18010619e-06,-8.03448196e-03, -1.31197493e-06, -8.48779891e-03, -  
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1.74850348e-03, -9.19962458e-07,1.62741486e-03, 5.35235376e-08, 1.10216831e-02];
```