

【fourier 級数】

$$\text{wave1: } f(t) = \begin{cases} 0 & (\pi < t < 0) \\ 1 & (0 < t < \pi) \end{cases}$$

fourier 係数

$$a_0 = \frac{1}{\pi} \int_0^\pi dt = 1, a_k = \frac{1}{\pi} \int_0^\pi \cos(kt) dt = 0$$

$$b_k = \frac{1}{\pi} \int_0^\pi \sin(kt) dt = \frac{1}{k\pi} (1 - \cos(k\pi)) = \frac{1}{k\pi} \{1 - (-1)^k\}$$

$$\blacksquare f(t) = \frac{1}{2} + \sum_{k=1}^{+\infty} \left[\frac{1}{k\pi} \{1 - (-1)^k\} \sin(kt) \right]$$

$$\text{wave2: } f(t) = \begin{cases} 0 & (\pi < t < 0) \\ t & (0 < t < \pi) \end{cases}$$

fourier 係数

$$a_0 = \frac{1}{\pi} \int_0^\pi t dt = \frac{\pi}{2}, a_k = \frac{1}{\pi} \int_0^\pi t \cdot \cos(kt) dt = \frac{(-1)^k - 1}{k^2\pi}$$

$$b_k = \frac{1}{\pi} \int_0^\pi t \cdot \sin(kt) dt = \frac{(-1)^{k+1}}{k}$$

$$\blacksquare f(t) = \frac{\pi}{4} + \sum_{k=1}^{+\infty} \left[\frac{(-1)^k - 1}{k^2\pi} \cos(kt) + \frac{(-1)^{k+1}}{k} \sin(kt) \right]$$

$$\text{wave3: } f(t) = |t| \quad (-\pi \leq t \leq \pi)$$

fourier 余弦級数 ($\because f(t) = \text{even function}$)

$$b_k = 0$$

$$a_0 = \frac{2}{\pi} \int_0^\pi t dt = \pi, a_k = \frac{2}{\pi} \int_0^\pi t \cdot \cos(kt) dt = \frac{2\{(-1)^k - 1\}}{k^2\pi}$$

$$\blacksquare f(t) = \frac{\pi}{2} + \sum_{k=1}^{+\infty} \left[\frac{2\{(-1)^k - 1\}}{k^2\pi} \cos(kt) \right]$$

$$\text{wave4: } f(t) = \frac{1}{2} t^2 \quad (-\pi \leq t \leq \pi)$$

fourier 余弦級数 ($\because f(t) = \text{even function}$)

$$b_k = 0$$

$$a_0 = \frac{2}{\pi} \int_0^\pi \frac{1}{2} t^2 dt = \frac{\pi^2}{3}, a_k = \frac{2}{\pi} \int_0^\pi t^2 \cdot \cos(kt) dt = \frac{2(-1)^k}{k^2}$$

$$\blacksquare f(t) = \frac{\pi^2}{6} + \sum_{k=1}^{+\infty} \left[\frac{2(-1)^k}{k^2} \cos(kt) \right]$$

$$\text{wave5: } f(t) = t \quad (-\pi < t < \pi)$$

fourier 正弦級数 ($\because f(t) = \text{odd function}$)

$$a_0, a_k = 0$$

$$b_k = \frac{2}{\pi} \int_0^\pi t \cdot \sin(kt) dt = \frac{2(-1)^{k+1}}{k}$$

$$\blacksquare f(t) = \sum_{k=1}^{+\infty} \left[\frac{2(-1)^{k+1}}{k} \sin(kt) \right]$$

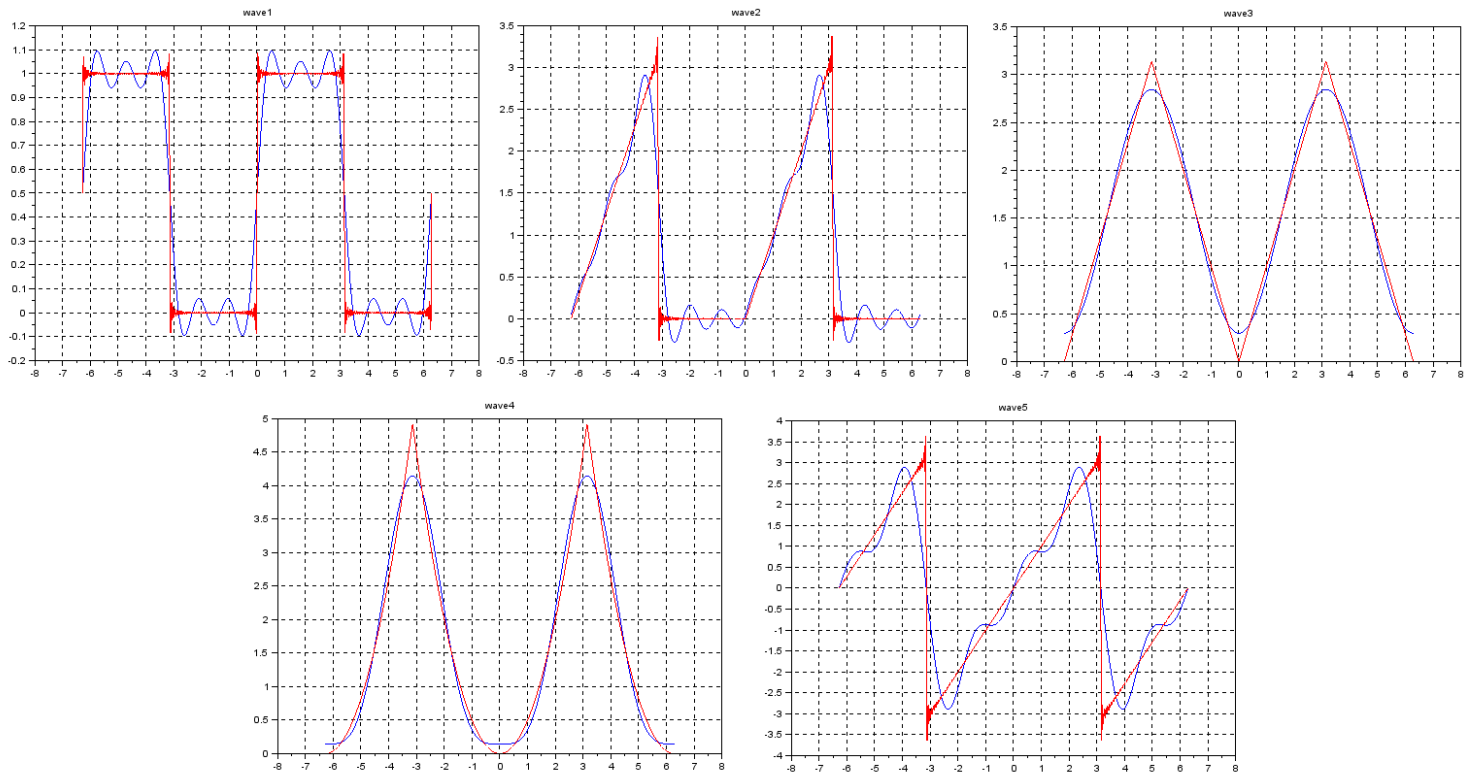


Figure 1: Scilab 実行結果

Source Code 1: Scilab

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////////////////////////////////////
//   フーリエ級数
//   fourier Series
//
//                               M.Tsutsui
////////////////////////////////////

////////////////////////////////////wave1////////////////////////////////////
clear;
pi=%pi;//円周率
t=linspace(-2*pi,2*pi,1000);//等間隔ベクトル作成(周期を決定)
n=100; //Σ項数を入力
ft=0;
for k=1:1:n
    ft=ft+(1/(2*n))+(((1-((-1)^k))/(k*pi)))*\sin(k*t);
end
plot(t,ft,'r');
plot(t,\sin(t),'k');
xgrid;

////////////////////////////////////wave2////////////////////////////////////
clear;
pi=%pi;//円周率
t=linspace(-2*pi,2*pi,1000);//等間隔ベクトル作成(周期を決定)
n=100; //Σ項数を入力
ft=0;
for k=1:1:n
    ft=ft+(pi/(4*n))+(((-1)^k-1)/((k^2)*pi)*\cos(k*t)-((-1)^k/k)*\sin(k*t);end
plot(t,ft,'r');
xgrid;

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//////////wave3//////////
clear;
pi=%pi;//円周率
t=linspace(-2*pi,2*pi,1000);//等間隔ベクトル作成(周期を決定)
n=100; //Σ項数を入力
ft=0;
for k=1:1:n
    ft=ft+(pi/(2*n))+2*((-1)^k-1)/((k^2)*pi)*cos(k*t);
end
plot(t,ft,'r');
xgrid;

//////////wave4//////////
clear;
pi=%pi;//円周率
t=linspace(-2*pi,2*pi,1000);//等間隔ベクトル作成(周期を決定)
n=100; //Σ項数を入力
ft=0;
for k=1:1:n
    ft=ft+((pi^2)/(6*n))+2*(-1)^k/(k^2)*cos(k*t);
end
plot(t,ft,'r');
xgrid;

//////////wave5//////////
clear
pi=%pi;//円周率
t=linspace(-2*pi,2*pi,1000);//等間隔ベクトル作成(周期を決定)
n=100; //Σ項数を入力
ft=0;
for k=1:1:n
    ft=ft+(2*(-1)^(k+1))/k*sin(k*t);
end
plot(t,ft,'r');
xgrid;
//end//////////

```