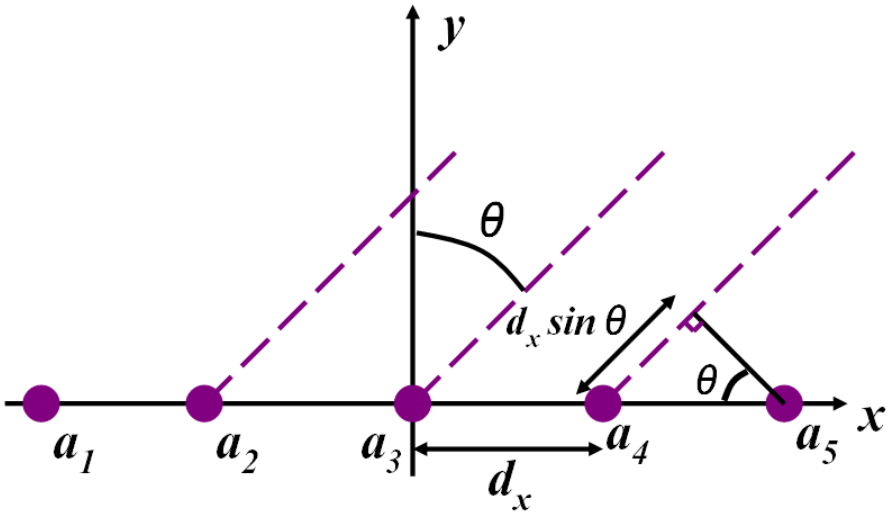
【アレイ信号処理・ビームフォーマ　　Beam Former】



s を音圧，ω を角周波数とすると，平面波は次のように表される．

$$s(\boldsymbol{p},t) = A\exp(j(\omega t - \boldsymbol{k}^T\boldsymbol{p})) = A\exp(j\omega t)\cdot\exp(-j\boldsymbol{k}^T\boldsymbol{p})$$

$\boldsymbol{p}$ は任意の観測点で，$\boldsymbol{p} = [p_x, p_y, p_z]$．$\boldsymbol{k}$ は波数ベクトル $\boldsymbol{k} = [k_x, k_y, k_z]$ で，A は定数である．
$A\exp(j\omega t)$ は信号源を表し，$\exp(-j\boldsymbol{k}^T\boldsymbol{p})$ は位相差を表す．

直線上等間隔アレイの位置ベクトル

$$\boldsymbol{p}_m = [(m-1) - \frac{M-1}{2})d_x,\ \ 0,\ \ 0]^T \quad m = 1, 2, \cdots, M \quad (M\text{:センサ数 },d_x\text{:アレイ間隔})$$

直線上アレイに対する m 番目の要素は，

$$a_m = \exp(-j\boldsymbol{k}^T\boldsymbol{p}_m) = \exp[-j((m-1) - \frac{M-1}{2})k_x d_x)] \quad k_x\text{:波数 } (= -\frac{2\pi}{\lambda}\sin\theta)$$

ビームパターンは，全要素の和をとり，

$$\Psi(k_x) = \frac{1}{M}(a_1 + a_2 + \cdots a_m) = \frac{1}{M}\sum_{m=1}^{M}\exp[-j((m-1) - \frac{M-1}{2})k_x d_x]$$

$$= \frac{1}{M}\cdot e^{j\frac{M-1}{2}k_x d_x}\sum_{m=1}^{M} e^{-j(m-1)k_x d_x} = \frac{1}{M}\cdot e^{j\frac{M-1}{2}k_x d_x}\cdot\frac{1-e^{-jMk_x d_x}}{1-e^{-jk_x d_x}} = \frac{1}{M}\cdot e^{j\frac{M-1}{2}k_x d_x}\cdot\frac{e^{-j\frac{Mk_x d_x}{2}}\cdot 2j\sin(\frac{Mk_x d_x}{2})}{e^{-j\frac{k_x d_x}{2}}\cdot 2j\sin(\frac{k_x d_x}{2})}$$

$$= \frac{1}{M}\cdot\frac{\sin(\frac{Mk_x d_x}{2})}{\sin(\frac{k_x d_x}{2})}$$

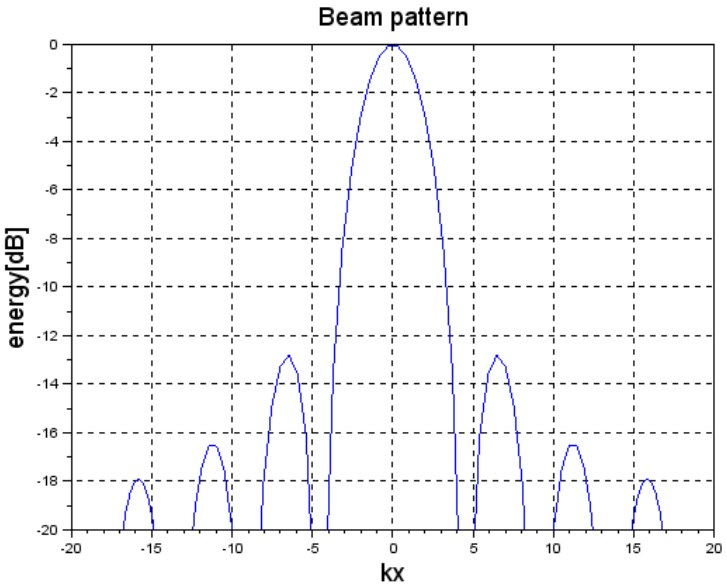


Figure 1: Scilab 実行結果

Source Code 1: Scilab

```
////////////////////////////////
//      ビームパターン
//      Beam pattern
//
//                    M.Tsutsui
////////////////////////////////

clear;

pi=%pi;
eps=%eps;

D_size=100;//データ数
T=25;//摂氏温度
c=331.5+0.61*T;//音速
f=1000;//周波数

sita_min=-pi/2 ; //下端
sita_max=pi/2; //上端
sita=linspace(sita_min,sita_max,D_size);//横軸指定
M=8; //直線状等間隔アレイ
lambda=c/f;//波長
dx=lambda/2;//アレイ間隔

kx=(-2*pi)/lambda.*sin(sita);//波数
pusi=(sin((M*kx*dx)./2))./(M*sin((kx*dx)/2)+eps);
pusi_log=20*log10(abs(pusi));//対数
plot(kx,pusi_log);
fg=gca();
fg.data_bounds(:,2)=[-20;0]; //縦軸範囲指定
xlabel("kx", "fontsize", 4);
ylabel("energy[dB]", "fontsize", 4);
xgrid();
title('Beam pattern','fontsize',4);
```

Source Code 2: Python

```
#--------------------------------
# Module_Name:Beam pattern
# Author: m_tsutsui
#--------------------------------

#Library_Import###############################
from numpy import*
from scipy import*
from pylab import *

import math, numpy as np
import matplotlib.pyplot as plt
#Library_Import_end###########################

if __name__ == "__main__":
    D_size=200 #Data_Number
    T=25 #temperature
    c=331.5+0.61*T #speed_of_sound
    f=1000 #Frequency
    lambda=c/f #Wave_Length
    dx=lambda/2 #Array_Interval

    M=9 #Array_Number
    sita_min=-pi/2 #Θ_min
    sita_max=pi/2 #Θ_max

    sita=linspace(sita_min,sita_max,D_size)#Theata

    kx=(-2*pi)/lambda*sin(sita) #Wave_Number
    pusi=(sin((M*kx*dx)/2))/(M*sin((kx*dx)/2)) #beam_pattern
    pusi_log=20*log10(abs(pusi))

#plot_command#############
    plt.figure(facecolor='w')#Backgroundcolor_white
    plt.plot(kx,pusi_log)
    ylim(ymin=-20)
    plt.xlabel('kx[1/m]',fontsize=15)
    plt.ylabel('Energy[dB]',fontsize=15)
    plt.grid()
    plt.title('Beam pattern')
    plt.show()
```

■ $M \cdot dx$ 変化 ビームパターン

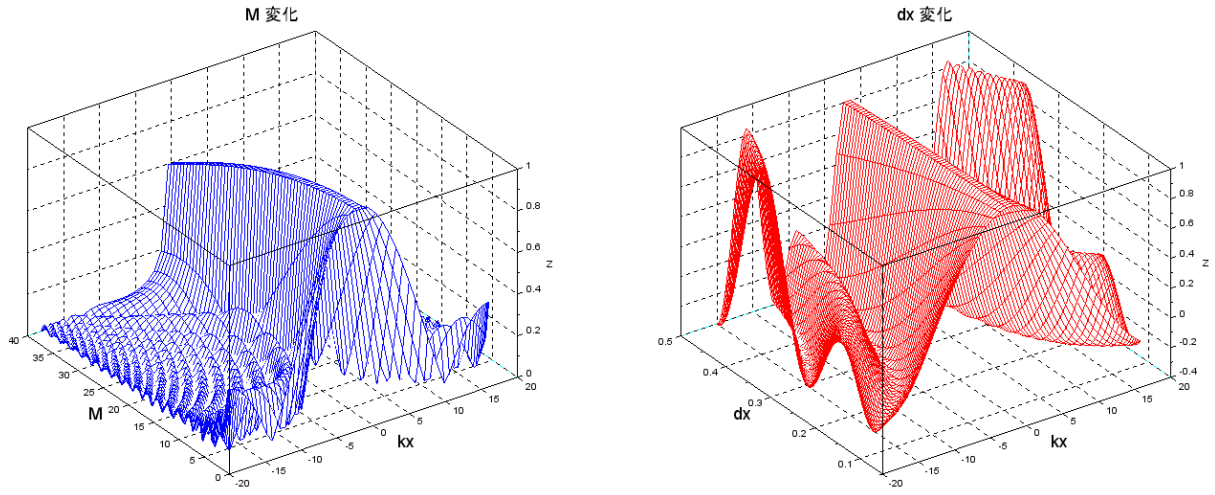


Figure 2: Scilab 実行結果

Source Code 3: Scilab

```
////////////////////////////////
//      kx-M Beam pattern
//
//                      M.Tsutsui
////////////////////////////////

clear all;

pi=%pi;
eps=%eps;

D_size=60;//データ数
T=25;//摂氏温度
c=331.5+0.61*T;//音速
f=1000;//周波数

sita_min=-pi/2 ; // θ 下端
sita_max=pi/2; // θ 上端
sita_p=linspace(sita_min,sita_max,D_size);// θ
M_min=3;
M_max=40;
M_p=linspace(M_min,M_max,D_size);//センサ数
[sita,M]=meshgrid(sita_p,M_p);//メッシュ作成

lambda=c/f;//波長
dx=lambda/2;//アレイ間隔_fix

kx=(-2*pi)/lambda.*sin(sita);//波数
pusi=(sin((M.*kx*dx)./2))./(M.*sin((kx*dx)/2)+eps);//M_sweep_dx_fix


surf(kx,M,abs(pusi),'facecol','w','edgecol','b');
xlabel("kx", "fontsize", 4);
ylabel("M", "fontsize", 4);
xgrid();
title('M␣変化','fontsize',4);
```

Source Code 4: Scilab

```
////////////////////////////////
//      kx-dx Beam pattern
//
//                      M.Tsutsui
////////////////////////////////

clear all;

pi=%pi;
eps=%eps;

D_size=60;//データ数
T=25;//摂氏温度
c=331.5+0.61*T;//音速
f=1000;//周波数
M=5;

sita_min=-pi/2 ; //下端
sita_max=pi/2; //上端
sita_p=linspace(sita_min,sita_max,D_size);// θ
lambda=c/f;//波長
dx_p=linspace(lambda/4,1.3*lambda,D_size);//dx_sweep

[sita,dx]=meshgrid(sita_p,dx_p);//メッシュ作成

kx=(-2*pi)/lambda.*sin(sita);//波数

pusi=(sin((M.*kx.*dx)./2))./(M.*sin((kx.*dx)./2)+eps);//dx_sweep_M_fix

surf(kx,dx,pusi,'facecol','w','edgecol','r');
xlabel("kx", "fontsize", 4);
ylabel("dx", "fontsize", 4);
zlabel("energy", "fontsize", 4);
xgrid();
title('dx␣変化','fontsize',4);
```

Source Code 5: Python

```
#-----------------------------------
# Module_Name:kx-M Beam pattern
# Author:m_tsutsui
#-----------------------------------

#Library_Import###############################
from numpy import*
from scipy import*
from pylab import *
from mpl_toolkits.mplot3d import Axes3D #3Dim_Plot

import math, numpy as np
import matplotlib.pyplot as plt
#Library_Import_end###########################


if __name__ == "__main__":
    T=25 #temperature
    c=331.5+0.61*T #the_speed_of_sound
    f=1000 #frequency
    lambda=c/f #wave_length
    dx=lambda/2 #array_interval_(fix)
    D_size=50 #Data_Number

    sita_min=-pi/2 #theta_min
```

```
    sita_max=pi/2 #theta_max
    sita_p=linspace(sita_min,sita_max,D_size) #direction
    M_min=3
    M_max=40
    M_p=linspace(M_min,M_max,D_size) #sensor_number
    sita,M=meshgrid(sita_p,M_p) #mesh

    kx=(-2*pi)/lambda*sin(sita) #wave_number

    pusi=(sin((M*kx*dx)/2))/(M*sin((kx*dx)/2)) #M_sweep_dx_fix

    aaaa=np.size(pusi,1)
#3Dim_Plot##############################

    fig = plt.figure(facecolor='w')
    ax = Axes3D(fig)
    ax.plot_wireframe(kx,M,abs(pusi),color = 'b',linewidth = 0.3)
    plt.xlabel('kx[1/m]',fontsize=15)
    plt.ylabel('M',fontsize=15)
    plt.title('kx-M Beam pattern')

    plt.show()
```

Source Code 6: Python

```
#-------------------------------------------------------------------
# Module_Name:kx-dx Beam pattern
# Author:m_tsutsui
#-------------------------------------------------------------------

#Library_Import#################################
from numpy import*
from scipy import*
from pylab import *
from mpl_toolkits.mplot3d import Axes3D #3Dim_Plot

import math, numpy as np
import matplotlib.pyplot as plt
#Library_Import_end#############################


if __name__ == "__main__":
    T=25 #temperature
    c=331.5+0.61*T #the_speed_of_sound
    f=1000 #frequency
    lambda=c/f #wave_length
    dx=lambda/2 #array_interval_(fix)
    D_size=60 #Data_Number


    M=5 #M_fix
    sita_min=-pi/2 #theta_min
    sita_max=pi/2 #theta_max
    sita_p=linspace(sita_min,sita_max,D_size) #direction
    dx_p=linspace(lambda/4,1.3*lambda,D_size) #dx_sweep
    sita,dx=meshgrid(sita_p,dx_p) #mesh

    kx=(-2*pi)/lambda*sin(sita) #wave_number

    pusi=(sin((M*kx*dx)/2))/(M*sin((kx*dx)/2)) #dx_sweep_M_fix
#3Dim_Plot##############################

    fig = plt.figure(facecolor='w')
    ax = Axes3D(fig)
```

```
ax.plot_wireframe(kx,dx,pusi,color = 'r',linewidth = 0.3)
plt.xlabel('kx[1/m]',fontsize=15)
plt.ylabel('dx',fontsize=15)
plt.title('kx-dx Beam pattern')

plt.show()
```

■方向-周波数応答

$$(1)\Psi(k_x) = \frac{1}{M} \cdot \frac{\sin(\frac{Mk_xd_x}{2})}{\sin(\frac{k_xd_x}{2})} \ , \quad (2)k_x = -\frac{2\pi}{\lambda}\sin\theta \ , \quad (3)\lambda = \frac{c}{f} \ (c\text{:音速})$$

(1)，(2)，(3) からビームフォーマの方向-周波数応答は，$\Psi(\theta,\omega) = \frac{1}{M} \cdot \frac{\sin(\frac{Md_x\omega}{2c}\sin\theta)}{\sin(\frac{d_x\omega}{2c}\sin\theta)}$

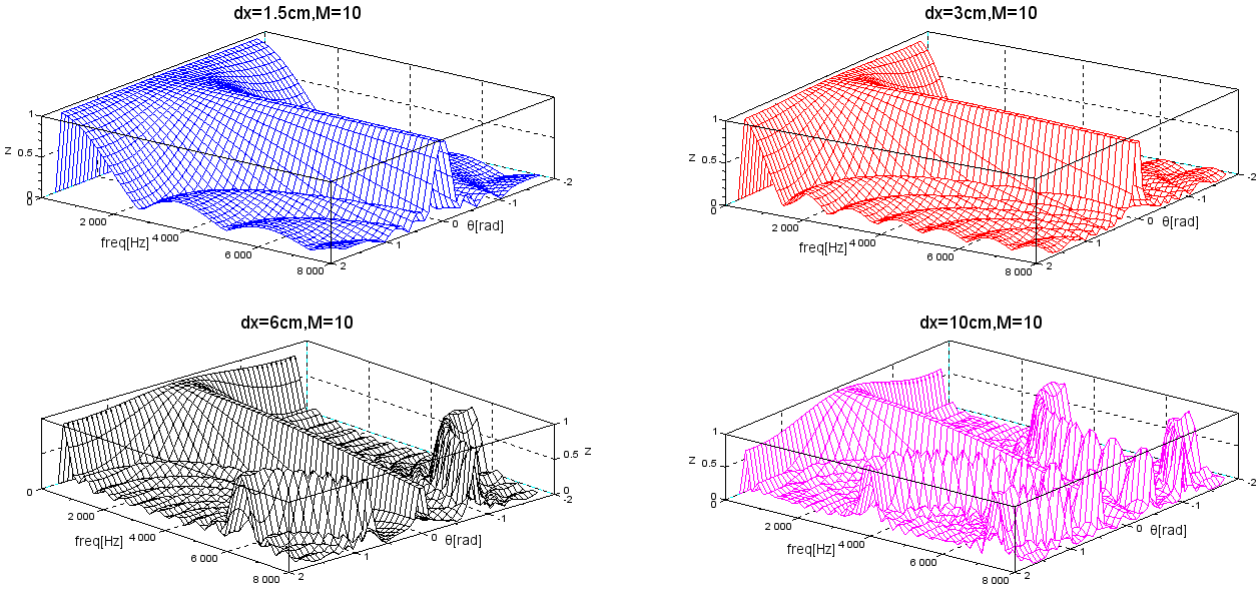


Figure 3: Scilab 実行結果

Source Code 7: Scilab

```
//////////////////////////////////////
//      方向-周波数 応答
//      direction-frequency response
//
//                          M.Tsutsui
//////////////////////////////////////

clear;

funcprot(0);
function[D_F_Res_o]=D_F_Res(dx,M);//関数定義
      D_F_Res_o=1/M*(sin(M*dx/2*omega./c.*sin(sita)))./(sin(dx/2*omega./c.*sin(sita))+eps);//ビームパターン
endfunction


pi=%pi;
eps=%eps;
```

```
D_size=50;//データ数
T=25;//摂氏温度
c=331.5+0.61*T;//音速

sita_min=-5/9*pi;//方向下限
sita_max=5/9*pi;//方向上限
f_min=0;//周波数下限
f_max=8000;//周波数上限

sita_p=linspace(sita_min,sita_max,D_size);//方向ベクトル
f_p=linspace(f_min,f_max,D_size);//周波数ベクトル
[sita,f]=meshgrid(sita_p,f_p);//メッシュ作成
omega=2*pi*f;//角周波数


subplot(2,2,1);
surf(sita,f,abs(D_F_Res(0.015,10)),'facecol','w','edgecol','b');//アレイ間隔1.5cm,要素数10
xgrid();
xlabel("θ [rad]", "fontsize", 3);
ylabel("freq[Hz]", "fontsize", 3);
title('dx=1.5cm,M=10','fontsize',4);

subplot(2,2,2);
surf(sita,f,abs(D_F_Res(0.03,10)),'facecol','w','edgecol','r');//アレイ間隔3cm,要素数10
xgrid();
xlabel("θ [rad]", "fontsize", 3);
ylabel("freq[Hz]", "fontsize", 3);
title('dx=3cm,M=10','fontsize',4);

subplot(2,2,3);
surf(sita,f,abs(D_F_Res(0.06,10)),'facecol','w','edgecol','k');//アレイ間隔6cm,要素数10
xgrid();
xlabel("θ [rad]", "fontsize", 3);
ylabel("freq[Hz]", "fontsize", 3);
title('dx=6cm,M=10','fontsize',4);

subplot(2,2,4);
surf(sita,f,abs(D_F_Res(0.1,10)),'facecol','w','edgecol','m');//アレイ間隔10cm,要素数10
xgrid();
xlabel("θ [rad]", "fontsize", 3);
ylabel("freq[Hz]", "fontsize", 3);
title('dx=10cm,M=10','fontsize',4);
```

Source Code 8: Python

```
#-----------------------------------------------
# Module_Name:direction-frequency response
# Author:m_tsutsui
#-----------------------------------------------

#Library_Import###############################
from numpy import*
from scipy import*
from pylab import *
from mpl_toolkits.mplot3d import Axes3D #3Dim_Plot

import math, numpy as np
import matplotlib.pyplot as plt
#Library_Import_end###########################

def D_F_Res(dx,M):#_define_beam_pattern_function
    return 1/M*(sin(M*dx/2*omega/c*sin(sita)))/(sin(dx/2*omega/c*sin(sita)))

if __name__ == "__main__":
```

```
    T=25 #temperature
    c=331.5+0.61*T #the_speed_of_sound
    D_size=50 #Data_Number

    sita_min=-5/9*pi
    sita_max=5/9*pi
    f_min=0 #freq_min
    f_max=8000 #freq_max

    sita_p=linspace(sita_min,sita_max,D_size) #direction
    f_p=linspace(f_min,f_max,D_size) #frequency
    sita,f=meshgrid(sita_p,f_p) #mesh
    omega=2*pi*f #angular_frequency

#3Dim_Plot############################

    fig = plt.figure(facecolor='w')
    ax = fig.add_subplot(2, 1, 1, projection='3d')
    ax.plot_wireframe(sita,f,abs(D_F_Res(0.03,10)),color = 'b',linewidth = 0.5)
    plt.xlabel('$\\theta$[rad]',fontsize=15)
    plt.ylabel('freq[Hz]',fontsize=15)
    plt.title('dx=3cm,M=10')


    ax = fig.add_subplot(2, 1, 2, projection='3d')
    ax.plot_wireframe(sita,f,abs(D_F_Res(0.06,10)),color = 'r',linewidth = 0.5)
    plt.xlabel('$\\theta$[rad]',fontsize=15)
    plt.ylabel('freq[Hz]',fontsize=15)
    plt.title('dx=6cm,M=10')


    plt.show()
```